

# PHY 101

## Applications of Newton's Laws

Outline

Particles in Equilibrium

Dynamics of particles

Frictional forces

Dynamics of circular motion

# Introduction

The Newton's laws of motion are the foundation of classical mechanics and it can be stated in a very simply manner. But applying these laws to real situations such as a toboggan sliding down a hill, or an airplane making a steep turn requires analytical skills and problem-solving technique.

- We'll begin with equilibrium problems, in which we analyze the forces that act on a body at rest or moving with constant velocity (Particles in Equilibrium);
- We'll then consider bodies that are not in equilibrium, for which we'll have to deal with the relationship between forces and motion (Dynamics of particles);
- We'll learn how to describe and analyze the contact force that acts on a body when it rests on or slides over a surface (Frictional forces); and
- We'll also analyze the forces that act on a body that moves in a circle with constant speed. We shall close the lecture with a brief look at the fundamental nature of force and the classes of forces found in our physical universe (Dynamics of circular motion).

# Particles in Equilibrium

A body under the influence of forces is called a particle. It is said to be at equilibrium when it is at rest or moving with constant velocity in an inertial frame of reference.

Examples: A hanging lamp, a kitchen table, an airplane flying straight and level at a constant speed.

The essential physical principle is Newton's first law: When a particle is in equilibrium, the net force acting on it—that is, the vector sum of all the forces acting on it—must be zero:

$$\sum \vec{F} = \mathbf{0} \quad (\text{particle in equilibrium, vector form})$$

$$\sum F_x = 0 \quad \sum F_y = 0 \quad (\text{particle in equilibrium, component form})$$

# Particles in Equilibrium

i Two 250 N weights are suspended at opposite ends of a rope which passes over a light frictionless pulley is attached to a chain which goes to the ceiling. Determine the tension in the chain.

## Solution:

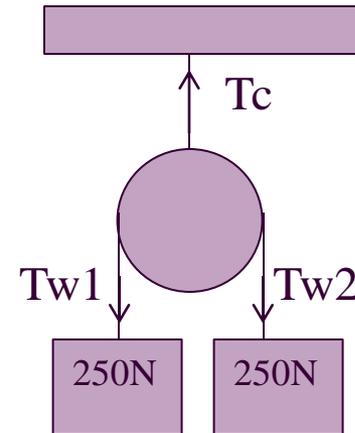
Let  $T_w$  be the tension cause by each weight and the  $T_c$  is the tension on the chain by the ceiling on the pulley.

The net force on the pulley is:

$$\longrightarrow T_c - (T_w1 + T_w2) = 0$$

$$T_c = T_w1 + T_w2 = 250 + 250$$

$$T_c = 500 \text{ N}$$



## Solution

The forces acting on the wight are shown in the second figure, For Equilibrium, vetical resolution of forces gives:

$$183 \cos 60^\circ + T \cos(90^\circ - \theta) - 250 = 0; T \sin \theta = 158.5$$

Horizontal resolution gives

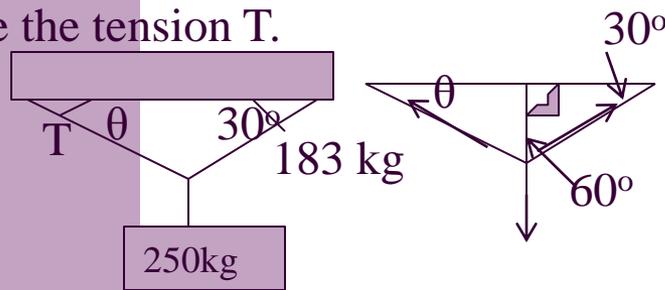
$$183 \sin 60^\circ + T \sin(90^\circ - \theta) - 250 = 0; T \cos \theta = 158.5$$

$$T^2 (\sin^2 \theta + \cos^2 \theta) = 158.5^2 + 158.5^2$$

$$T = 224.1 \text{ kg}; \theta = \tan^{-1}(158.8/158.5) = 45^\circ.$$

ii A wight of 250 kg is suspended vertically from two strings as shown below at point O. If the tension in the right string is 183 kg.

Determine the tension T.



# Dynamics of particles

A body is said to be dynamics when the net force is NOT equal to zero. This is application of Newton's second law in which the velocity of bodies in motion is not constant. These bodies are not in equilibrium and hence are accelerating. The net force on the body is equal to the mass of the body times its acceleration:

$$\sum \vec{F} = m\vec{a} \quad (\text{Newton's second law, vector form})$$

$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad (\text{Newton's second law, component form})$$

You have to use Newton's second law for any problem that involves forces acting on an accelerating body.

# Impulse and Momentum

- An impulse is the product of the force and the time in which the force acts while the momentum is the product of the mass and the velocity of the body.
- A body that increases its velocity from  $u$  to  $v$  within an interval of time  $t$  acquires an acceleration:  $a = \frac{v-u}{t}$

» and the force acting on the body is:  $F = \frac{m(v-u)}{t}$

»

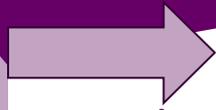
» Therefore,  $Ft = mv - mu$

- » The above equation states that the impulse on a body is equal to the change in momentum of the body.
- » Considering a body A of mass  $m_A$  with initial velocity  $u_A$  and collides with a second body B of mass  $m_B$  and initial velocity  $u_B$ . if the velocities of A and B after collision are  $v_A$  and  $v_B$  respectively,

# Solved problems

Then,

Initial momentum of A and B is equal to final momentum of A and B

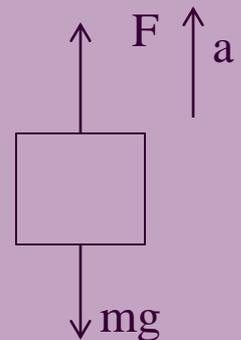


$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

This is the principle of conservation of momentum which states that, the total momentum of a system remain constant if no external force act on the system.

Example 1. An upward force of  $1.2 \times 10^4 \text{ N}$  acts on an elevator of mass  $2.0 \times 10^3 \text{ kg}$ . Calculate the elevator's acceleration.

Solution



The forces acting on the elevator are its weight,  $mg$ , downwards and upward force  $F$ . The net force is

$$F - mg = ma$$

$$1.2 \times 10^4 - (2.0 \times 10^3 \times 9.8) = (2.0 \times 10^3)a$$

$$a = -3.8 \text{ m/s}^2$$

# Frictional forces

Whenever two bodies interact by direct contact (touching) of their surfaces, we describe the interaction in terms of contact forces. The normal force is one example of a contact force; in this section we'll look in detail at another contact force, the force of friction.

Friction is important in many aspects of everyday life. The oil in a car engine minimizes friction between moving parts, but without friction between the tires and the road we couldn't drive or turn the car. Air drag—the frictional force exerted by the air on a body moving through it—decreases automotive fuel economy but makes parachutes work.

Without friction, nails would pull out and light bulbs would unscrew effortlessly.



## **Kinetic and Static Friction**

When you try to slide a heavy box of books across the floor, the box doesn't move at all unless you push with a certain minimum force. Then the box starts moving, and you can usually keep it moving with less force than you needed to

# Friction

- When you slide a heavy box across the floor
- You have to push with a strong enough force to get it moving.
- Once it is moving, you can keep it moving with less force than it took to get it started.
- If you make the box lighter, you need less force than before to start it and to keep it moving.

# Forces from a surface

- When an object rests or slides on a surface,
- The surface exerts a normal force perpendicular to the surface
- And a frictional force parallel to the surface.
- The direction of the frictional force is such that it always opposes the relative motion of the two surfaces.

# Kinetic Friction

- The frictional force that acts when a body slides over a surface is called kinetic friction.
- Its magnitude is given by:

$$f_k = \mu_k F_N$$

Magnitude of normal force

Coefficient of kinetic friction

# Static Friction

- The frictional force that acts on a body that is resting on a surface is called static Friction.
- Static friction is what resists the beginning of motion and makes it hard to get a box started sliding across the floor.
- The magnitude of static friction force is:

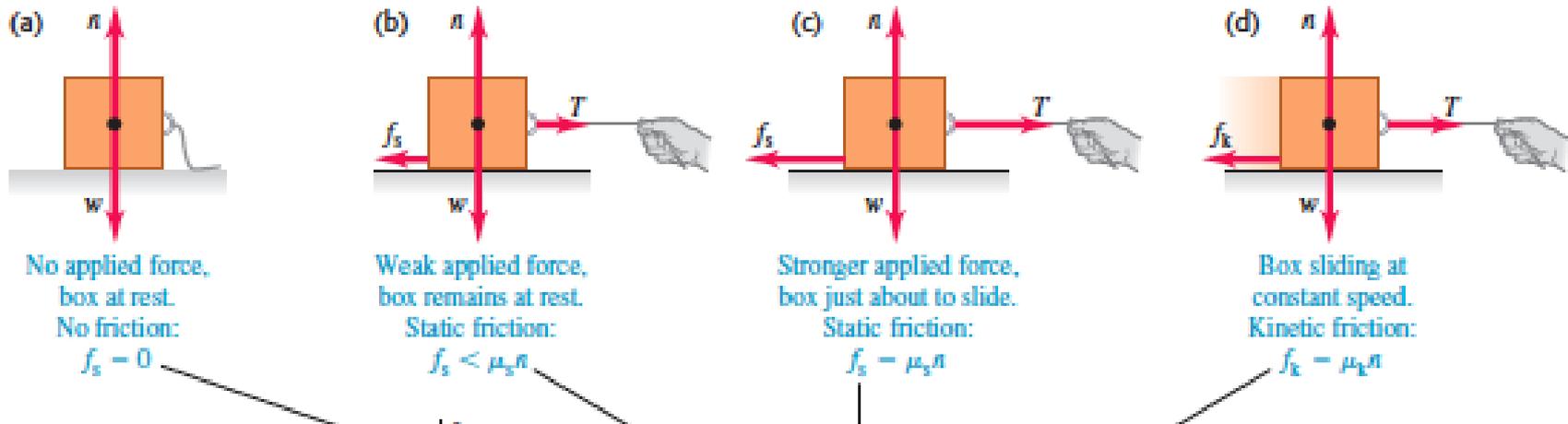
$$f_s = \mu_s F_N$$

Magnitude of normal force

Coefficient of static friction

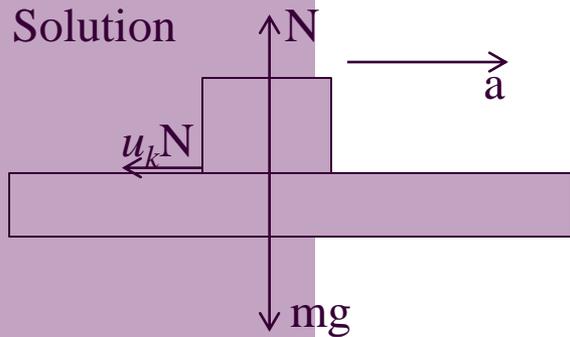
# Frictional forces

**5.19** (a), (b), (c) When there is no relative motion, the magnitude of the static friction force  $f_s$  is less than or equal to  $\mu_s n$ . (d) When there is relative motion, the magnitude of the kinetic friction force  $f_k$  equals  $\mu_k n$ . (e) A graph of the friction force magnitude  $f$  as a function of the magnitude  $T$  of the applied force. The kinetic friction force varies somewhat as intermolecular bonds form and break.



Example 1: A box of books of mass 2.0 kg is sliding across a level floor and its retardation is measured to be 4.0 m/s<sup>2</sup>. Calculate the coefficient of kinetic friction.

Solution



Resolving vertically gives  $N = mg$

The only horizontal force on the book is  $-u_k N = ma$ ,

$a = 4.0 \text{ m/s}^2$ ; Dividing equation 1 by 2

$$u_k = a/g = 4.0/9.8 = 0.408$$

# Dynamics of circular motion and gravitations

- Centripetal Acceleration: Many rigid bodies move in a circular path such as car negotiating a bend; the revolution of the satellites around the earth and the revolution of the planets around the sun. The acceleration of the circular moving body is directed radially towards the center of the circle and its magnitude is  $a_c = \frac{v^2}{r}$ ; where  $r$  is the radius of the path. From Newton's 2<sup>nd</sup> law,  $F_c = ma_c = \frac{mv^2}{r}$
- Periodicity of the uniform acceleration: This is the time ( $T$ ) taken for the particle to complete one revolution around the circular path of radius  $r$ . This is equal to the circumference of the circle divided by the speed.

$$T = \frac{2\pi r}{v}$$

$$a_{\text{rad}} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$$

Newton's law of gravitation: This states that the force of attraction of two bodies in the universe is proportional to the product of their masses and inversely proportional to the square of the distance separating their centers.

$$F_g = \frac{Gm_1m_2}{r^2}$$

Gravitational Potential Energy  $V_{(E)}$ : This is the work done in moving a body initially separated from the center of the earth by distance  $r$  is moved further distance  $dr$ .

$$dW = F_g dr$$

$$W_{grav} = \int_{r_1}^{r_2} f_g \cdot dr = \int \frac{GmM_E}{r^2} dr \quad \longrightarrow \quad V_{(E)} = -\frac{GM_E m}{r}$$

Thus a body on the surface of the Earth has a gravitational potential energy of  $-\frac{GM_E m}{r}$  while one at infinite distance from the earth zero gravitational potential energy

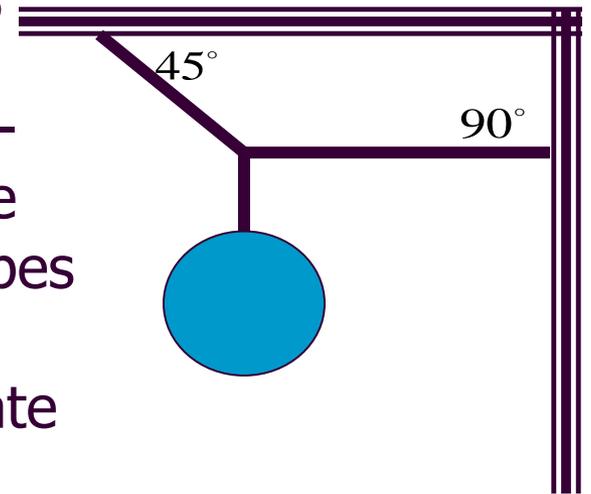
# Recap

- The coefficients of friction, both static and kinetic, have no units.
- The coefficients of friction must be less than 1.
- When the net force on a particle is zero
  - The particle is in equilibrium.
  - It has a constant velocity.
  - It has zero acceleration.
- When the net force on a particle is not zero
  - The particle is dynamic
  - The velocity is not constant
  - The particle possesses acceleration
- In uniform circular motion,
  - the acceleration vector is directed toward the center of the circle. The motion is governed by Newton's second law

$$a_{\text{rad}} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$$

# Assignment

1. A 52-N sled is pulled across a cement sidewalk at constant speed. A horizontal force of 36 N is exerted. What is the coefficient of kinetic friction between the sidewalk and the metal runners of the sled?
2. The sled from the example is now on packed snow. The coefficient of friction is 0.12. If a person weighing 650 N sits on the sled, what force is needed to slide the sled across the snow at a constant speed?
3. A 100-N body is shown suspended from a system of cords. What is the tension in the horizontal cord?
4. A skier of mass 65.0 kg is pulled up a snow-covered slope at constant speed by a tow rope that is parallel to the ground. The ground slopes upward at a constant angle of  $26^\circ$  above the horizontal and you can ignore friction. Calculate the tension in the tow rope.



5. A man is dragging a trunk up the loading ramp of a mover's truck. The ramp has a slope angle of  $20.0^\circ$  and the man pulls upward with force  $F$  whose direction makes an angle of  $30.0^\circ$  with the ramp. How large a force  $F$  is required in order for the component  $F_x$  parallel to the ramp to be 28.0 N? How large will the component  $F_y$  then be?

6. A 20-kg box rests on a frictionless ramp with a  $15^\circ$  slope. A mover pulls up on a rope attached to the box to pull it up the incline. If the rope makes an angle of  $40^\circ$  with the horizontal, what is the force the mover must exert on the box to give it an acceleration of  $1\text{m/s}^2$ ?

7. A 5-kg block is held at rest against a vertical wall by a horizontal force. What is the minimum horizontal force needed to prevent the block from falling if the coefficient of friction between the wall and the block is  $\mu_s=0.40$ ?