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Symbolic Logic

(Signs)

This is a process of using symbols to represent propositions or statements. Symbolic logic involves general use of symbols rather than propositions. Such symbols are:

For Logical propositions P, Q, R, S

for Connectives i.e. joining propositions, or showing relations b/w two propositions

- (\Rightarrow) implication e.g. $P \Rightarrow Q$, i.e. P implies Q
- (\wedge) conjunction e.g. $P \wedge Q$, i.e. P and Q
- (\neg) Negation e.g. $\neg P$, i.e. not P
- (\vee) Alternative e.g. $P \vee Q$, i.e. either P or Q
- (\equiv) Equivalence e.g. $P \equiv Q$, i.e. P is equal to Q
- ($\neg(\cdot)$) Disjunction e.g. $\neg(P \wedge Q)$ i.e. neither P and Q

Let us look at a syllogism

All men are mortal = All S is P or SaP
John is a man = R is S or RiS
Therefore John is mortal = $\therefore R$ is P or RiP

All men are mortal can be symbolised as = All S is P or SaP ; S stands for all men, while P stands for mortal. SaP means that it is a universal affirmative because a symbolises a universal affirmative proposition.

The sign \Rightarrow is called a sign of implication that is to say when you see S , you expect P .

All S is P (All men are mortal)

What is a Proposition

A proposition is a statement or argument of fact that is either true or false. A proposition is a statement that is verified as either True or False. A proposition or statement is a verbal or written declarative sentence that is either true or false.

Nwala (1997) "It usually contains two terms; a subject term and a predicate term joined by a copula (verb). The first term is the subject of which something is asserted. The second term is called a predicate. It asserts or says something about the subject."

Example: ^A My mother is a woman = T
Obi is a boy = T
 $10 + 10 + 5 = 25$ = T
Nigerians are black = T
The earth is a planet = T

^B My father is a handsome man = F
 $11 + 3 + \sqrt{4} + 8 = 22$ = F
 $3 + 4 + 5 = 8$ = F
Hurrah! = Exclamation = F
The moon is a satellite of the Sun = F

A truth statement has a truth value T, while a false statement has a truth value F.

There are three terms in a syllogism (subject, predicate and a verb) Example in the statement "Nigerians are black", the subject is Nigerians, the predicate is black, while the verb is are.

In the statement $10 + 10 + 5 = 25$, the subject is $10 + 10 + 5$, the predicate is 25, while the verb is =

Hurray! is an exclamation, not a ~~true~~ statement because it has no subject, predicate or verb.

Are you a Nigerian? is a question and not a declarative sentence hence is a statement or proposition.

The moon is a satellite of the sun. This statement or proposition has a truth value of F though is a proposition but because science proved that the moon is the satellite of the earth. It earned this statement the truth value of F.

$3 + 4 + 5 = 8$ This statement has a truth value of F because the summation of $3 + 4 + 5 = 12$ and not 8.

Types of Proposition

There are different types of proposition. The main types of propositions are:

Universal Affirmative = A All S is P = Sap

Universal Negative = E No S is P = Sep

Particular Affirmative = I Some S are P = Sip

Particular Negative = O Some S are not P = Sop

All animal Yorubas are Nigerian Sap = A

No Yorlubo is a white-man Sep = E

Some birds are beautiful. $SIP = 1$
No John is. Some students are not intelligent.

Quantifiers or Distributors

A statement is quantified or distributed when the terms in the proposition has relation between the subject and predicate. All men are mortal or All Nigerians are ^{mortal} animals. These two propositions are universal affirmative. A universal statement makes claim about every member of its subject. The universal affirmative proposition "All Nigerians are ^{mortal} animals" means:

If you are a Nigerian, then you are mortal

Negative Affirmative distribution.

No Nigerian is a white-man. The word "no" is distributed or quantified Nigerian to white-man.

Part of the job of the distributor or quantifier is to determine the degree or extent of distribution or relationship. In the statement "Some dogs are not trainable" shows that some dogs are trainable while some are not. $SIP = 2$ This is particular negative.

The statement student who read well make first class is SIP Particular affirmative.

Rules governing Syllogism

There are five major rules that enable us recognise a syllogistic argument.

- a) Rules relating to Quantity
- b) Rules relating to Quality

Rules relating to Quantity

- 1) Middle terms must be distributed at least once.

Example: All men are mortal

John is a man

Therefore John is mortal

The middle term is the term that occurs twice in the argument, often in the two premises of the syllogism. The term man is the middle term here.

- 2) No term is to be distributed in the conclusion which is not distributed in the premise

Yemi is a beautiful girl

Yemi is fat

Therefore Yemi is good

3 Rules relating to Quality

- 3) When both premises are negative, no conclusion can validly follow. Example: No human being is mortal

Dogs are ^{not} human being

Therefore dogs are mortal

This argument did not follow because the conclusion is not the consequence of the premises.

- 4) When the premises is negative, the conclusion must be negative. Example: No human being is mortal

John is a human being

Therefore, John is immortal

5) If both premises are affirmative the conclusion must be affirmative. Example: All Nigerians are black
Taiwo is a Nigerian
Therefore Taiwo is a black.

Use of Symbols or Signs in Logic (Symbolic Logic)

Propositions, statement or arguments can be represented with symbols as in mathematics.

Simple statements or propositions are represented by the letter P, Q, R, S, \dots which are called propositional variables. These variables are attached true or false values. These variables represent ^{logical} propositions.

There are other symbols known as **Connectives**.

Connectives join or link propositions, they show logical links between ^{any} two propositions. They are represented thus

Connective word	Symbol	Compound statement formed	Symbolic form
Not	\sim	Conjunction	$\sim p$
And	\wedge	Disjunction	$p \vee q$
OR	\vee	Implication	$p \Rightarrow q$
If then	$\equiv, \Rightarrow, \supset$	Equivalence	$p \Leftrightarrow q$
If and only if			

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Implication

Conjunction e.g. $P \supset Q$, that is P implies Q

Conjunction e.g. $P \cdot Q$ that is P and Q

Negation e.g. $\neg P$ that is not $\neg P$

Alternative e.g. $P \vee Q$ that is P or Q

Equivalence e.g. $P \equiv Q$ that is P is equal to Q

Disjunction e.g. $\neg(P \cdot Q)$ that is neither P nor Q

~~Implication~~ Implication

If today is Sunday, then my mother will cook rice.
This is $P \supset Q$. If it is Sunday P , then \supset mother will cook rice Q (Implication)

Conjunction

It is Sunday and mother cooked rice P and Q
It is Sunday (P) (\cdot) mother cooked rice (Q)
It is not (Conjunction)

Today is not Sunday ($\neg P$), mother did not cook rice
 $\neg P$ (Negation)

Either that today is Sunday or mother will cook rice
($P \vee Q$) Disjunctive alternative

It is not that today is Sunday, mother cooked rice
Disjunctive

Compound Proposition

Conjunction, Disjunction, alternative, implication

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Disjunction Syllogism

$\neg(p \cdot q)$ neither p and q

It is not the case that today is a lecture day and Taiwo is gone at home. ($p \cdot q$). It is a lecture day p . Therefore, Taiwo is not at home $\neg q$

$$\neg(p \cdot q)$$

$$p$$

$$\therefore \neg q$$

There is two valid arguments in a disjunction. When one is affirmed, the other is denied

$$\neg(p \cdot q)$$

$$p$$

$$\therefore \neg q$$

$$\neg(p \cdot q)$$

$$q$$

$$\therefore \neg p$$

Alternative Syllogism

Either today is a lecture ^{day} or Taiwo is at home ($p \vee q$). Today is not a lecture day, $\neg p$. Therefore Taiwo is at home

$$p \vee q$$

$$\neg p$$

$$\therefore q$$

We have two valid arguments in an alternative syllogism. Affirm either and deny the other consequence or antecedence.

$$p \vee q$$

$$\neg p$$

$$\therefore q$$

$$p \vee q$$

$$\neg q$$

$$\therefore p$$

Question _____
Write on both sides of the paper

Do not write
in either
margin

Conjunction

It is a lecture day and Taiwo is gone to lecture
(p. q). It is a lecture day p. Therefore, Taiwo is gone
to lecture q. (p. q)

p

$\therefore q$

Hypothetical, Alternative and Disjunctive Syllogism

from
Hypothetical Syllogism: This type of argument employ
"if - then" propositions or conditional propositions. It
is an argument in which something is said to be the case
on condition that another thing obtains. And if that first
thing (usually called the antecedent) obtains, then we assert
that second thing (called, the consequent). But we cannot
assert the antecedent because the consequent is
true

Example:

Hypothetical Proposition. If it rains students read
their courses well, they will pass their examination

i.e. if p then q or $p \supset q$

Hypothetical Argument. If Taiwo read well, he
will pass his JAMB. Taiwo read well, \therefore Taiwo passed
JAMB

$p \supset q$

p

$\therefore q$

In this argument, once we assert the antecedent,
the consequent follows. It is called *modus ponendo ponens*

How it follow that if the consequent, we
deny the antecedent. It is called *Modus Ponendo
Tollens*

However, the proposition if P then Q has the following alternative meanings.

- P implies Q
- P is sufficient for Q
- Q is necessary for P
- P only if Q
- Q if P
- Q follows P
- Q is consequent of P

TRUTH TABLES

The validity of arguments can be shown by assigning truth values (T or F, i.e. True or False) to the constituent propositions. Between two propositions there are only four possible truth relations. For example, P and Q

	P	Q
Both is true	T	T
P is true and Q is false	T	F
P is false and Q is true	F	T
Both are false	F	F

If we relate the truth relations of propositions to their logical relations (conjunction, disjunction, alternative implication, etc) we have the following truth tables.

P	Q	$P \cdot Q$ Conjunction	$P \vee Q$ Alternative Disjunction	$P \supset Q$ Implication	$\neg(P \cdot Q)$ Disjunction
T	T	T	T	T	F
T	F	F	T	F	T
F	T	F	T	T	T
F	F	F	F	T	T

Truth table for conditional proposition

P	Q	$P \vee Q$
T	T	T
T	F	F
F	T	T
F	F	T